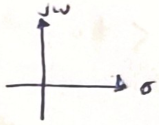
Introduction:-

» Laplace Transformation



$\mathcal{L}[f(t)] = F(s)$ ~~$= \int e^{st}\, dt\, [f(t)]$~~

complex variable $\sigma + j\omega$

$$= \int_0^\infty f(t)\, e^{-st}\, dt.$$

fun. of time

$$\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{c-j\omega}^{c+j\omega} F(s)\, e^{st}\, ds \quad , \quad t > 0$$

→ (not used ?!)

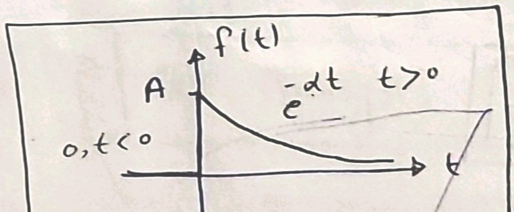
$$F(s) = \int_0^\infty f(t) e^{-st}\, dt = \text{converges}$$

(improper integral)

- $\mathcal{L}[A f(t)] = A F(s)$
- $\mathcal{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$

» Some frequently encountered time functions :-

[1] Exponential function :



$$f(t) = \begin{cases} A e^{-\alpha t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

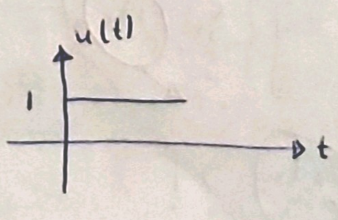
$$\mathcal{L}[f(t)] = \int_0^\infty A e^{-\alpha t} e^{-st}\, dt = A \int_0^\infty e^{-(\alpha+s)t}\, dt$$

$$= \frac{A}{s+a} \quad \text{(complex plane)}$$

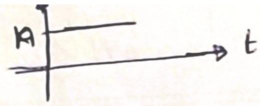
[2] The step function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

unit step function

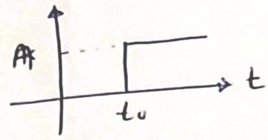


★ $Au(t) = \begin{cases} A & t > 0 \\ 0 & t < 0 \end{cases}$

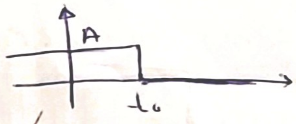


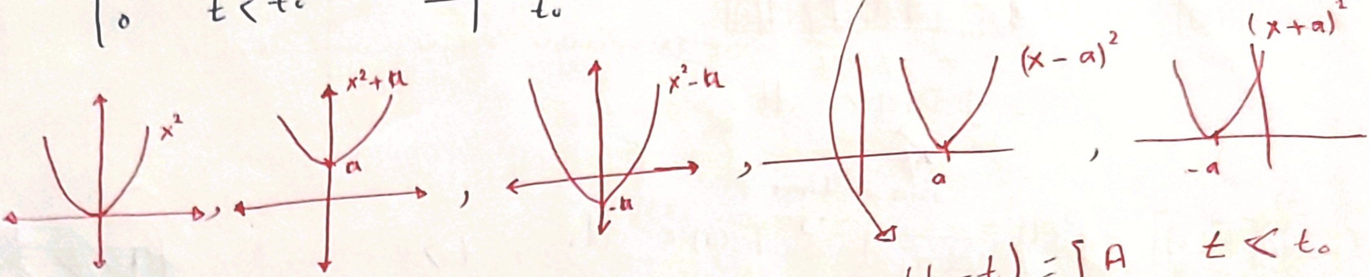
$A = 1$ unit step function

$Au(t-t_0) = \begin{cases} A & t > t_0 \\ 0 & t < t_0 \end{cases}$

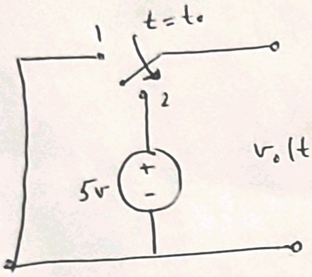


$A\,u(t_0 - t)\ ,\ t_0 > 0$

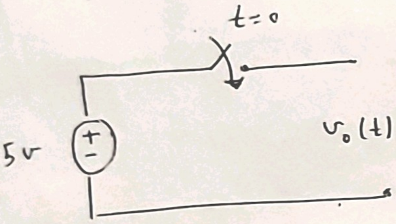




$Au(t_0 - t) = \begin{cases} A & t < t_0 \\ 0 & t > t_0 \end{cases}$



$v_0(t) \Rightarrow v_0(t) = 5u(t - t_0)$



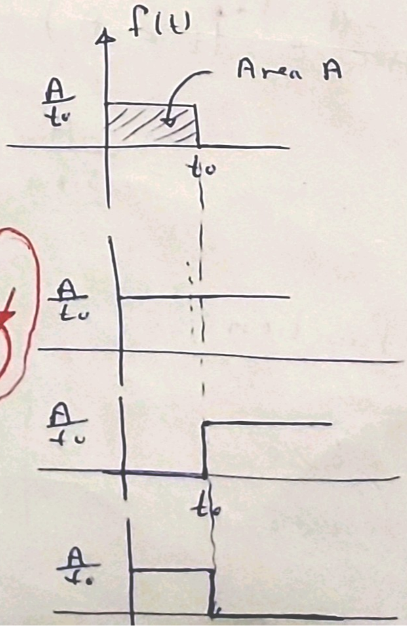
$v_0(t) \Rightarrow v_0(t) = 5u(t)$

$$\mathcal{L}[A] = \int_0^\infty A e^{-st}\, dt = \frac{A}{s}$$

$$\mathcal{L}[u(t)] = \boxed{\frac{1}{s}}$$

3) Pulse function

$$f(t) = \begin{cases} \frac{A}{t_0} & 0 < t < t_0 \\ 0 & t < 0,\ t > t_0 \end{cases}$$



$$f(t) = \frac{A}{t_0} u(t) - \frac{A}{t_0} u(t - t_0)$$

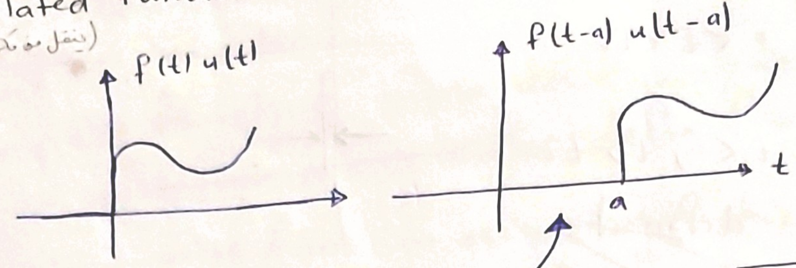
$$\mathcal{L}[f(t)] = \mathcal{L}[\quad] - \mathcal{L}[\quad]$$

e^{-st}
$e^{-s(t-t_0)}$
$e^{-st} \cdot e^{st_0}$

$$= \frac{A}{t_0} \cdot \frac{1}{s} - \frac{A}{t_0} \cdot \frac{1}{s} \cdot e^{-st_0} \quad ?!$$

* Translated function :-

(ينقل موقعات الدالة)



$\tau = t - a$

$$\mathcal{L}[f(t-a)u(t-a)] = \int_0^\infty f(t-a)\,u(t-a)\,e^{-st}\,dt$$

$$= \int_{-a}^{\infty} f(\tau)\,u(\tau)\,e^{-s(\tau+a)}\,d\tau$$

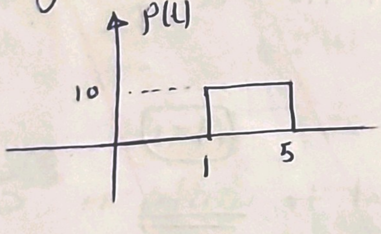
$$= \int_0^\infty f(\tau)\,u(\tau)\,e^{-s(\tau+a)}\,d\tau$$

$$= \int_0^\infty f(\tau)\,e^{-s\tau}\cdot e^{-as}\,d\tau = e^{-as}\int_0^\infty f(\tau)\,e^{-s\tau}\,d\tau$$

$$\boxed{\mathcal{L}[f(t-a)u(t-a)] = e^{-as}F(s)}$$

* example

rectangular pulse function



useful :-

CKT Analysis

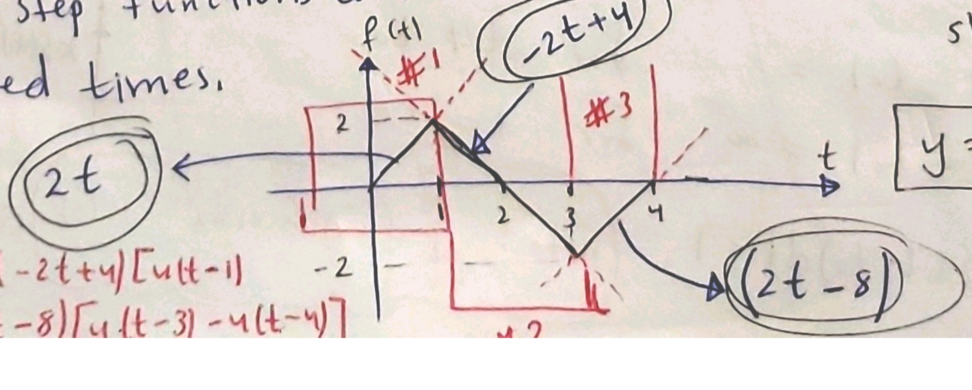
$p(t) = 10\,u(t-1) - 10\,u(t-5)$

find $\mathcal{L}[p(t)]$?!

* we use step functions to turn on ~~and~~ turn off linear function at desired times.

slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\boxed{y = m(x - x_0) + y_0}$$

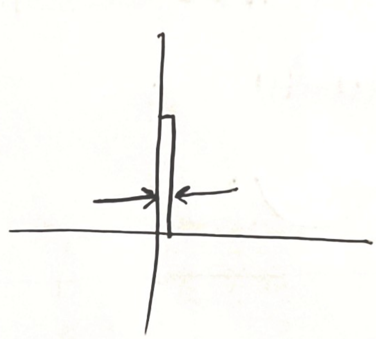


$2t[u(t) - u(t-1)] + (-2t+4)[u(t-1) - u(t-3)] + (2t-8)[u(t-3) - u(t-4)]$

4) The Impulse Function

$$g(t) = \lim_{t_0 \to 0} \frac{A}{t_0} \quad \text{(height)} \qquad 0 < t < t_0$$

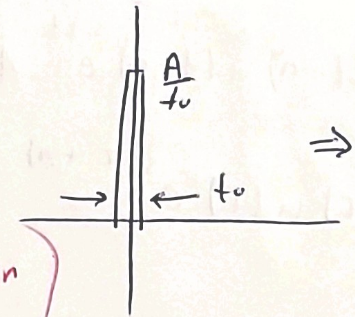
$$= 0 \qquad t < 0,\ t > t_0$$



The impulse function is defined:

$$\int_{-\infty}^{\infty} A\,\delta(t)\,dt = A$$

$$\delta(t) = 0 \qquad t \neq 0$$



$\Rightarrow$ area = A

Mathematically, the impulse function is defined

$$\int_{-\infty}^{\infty} k\,\delta(t)\,dt = k$$

$$\delta(t) = 0, \quad t \neq 0$$

$t_0 \to 0$

$\frac{A}{t_0} \to \infty$

area = A still

$$\mathcal{L}[g(t)] = \lim_{t_0 \to 0} \left[\frac{A}{t_0 s} \left(1 - e^{-s t_0}\right) \right]$$

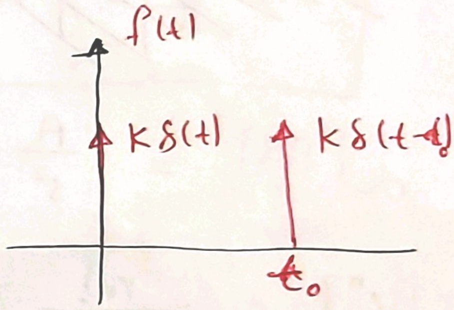
$$= \lim_{t_0 \to 0} \frac{\frac{d}{dt_0}\left[A\left(1 - e^{-s t_0}\right)\right]}{\frac{d}{dt_0}(t_0 s)} = \frac{As}{s} = A \quad \text{Area}$$

$e^{-st} = 1$

or $\mathcal{L}\{\delta(t)\} = \int_0^{\infty} \delta(t)\, e^{-st}\, dt = \int_0^{\infty} \delta(t)\, dt = 1 \leftarrow \leftarrow \leftarrow$

- The Laplace transform of the impulse function is equal to the area under the impulse.
- The impulse function whose area is equal to unity is called unit-impulse function ≡ Dirac delta function.
- The unit-impulse function occurring at $t = t_0$ is usually denoted by $\delta(t - t_0)$

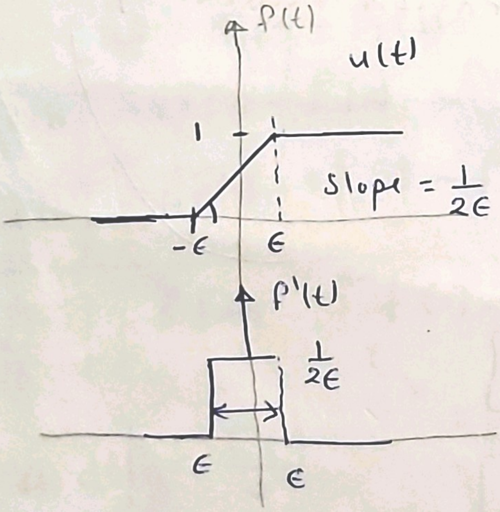
$$\begin{cases} \delta(t - t_0) = 0 & \text{for } t \neq t_0 \\ \delta(t - t_0) = \infty & \text{for } t = t_0 \\ \int_{-\infty}^{\infty} \delta(t - t_0)\, dt = 1 \end{cases}$$



⟹

$$\int_{-\infty}^{\infty} f(t)\, \delta(t)\, dt = f(0)$$

$$\int_{-\infty}^{\infty} f(t)\, \delta(t-a)\, dt = f(a)$$



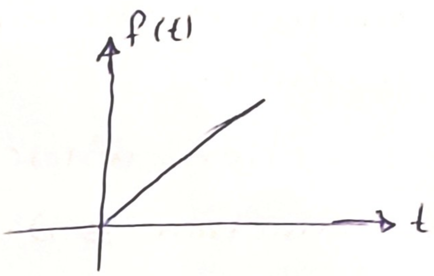
$f(t) \rightarrow u(t)$ as $\epsilon \rightarrow 0$

$f'(t) \rightarrow \delta(t)$ as $\epsilon \rightarrow 0$

$$\delta(t) = \frac{du(t)}{dt}$$

5 Ramp function

$$f(t) = \begin{cases} At & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$



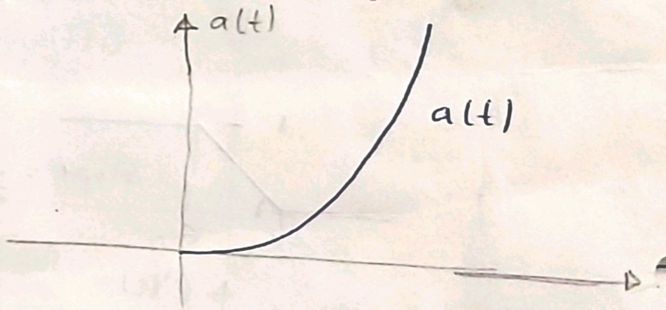
$$\mathcal{L}[At] = \int_0^\infty A t e^{-st} dt$$

(by parts)

$$\mathcal{L}[At] = \frac{A}{s^2}$$

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

6 unit-parabolic function (acceleration function)



$$a(t) = \frac{t^2}{2} \quad t \geq 0$$
$$= 0 \quad t < 0$$

$$\mathcal{L}\{a(t)\} = \mathcal{L}\left\{\frac{t^2}{2}\right\} = \frac{1}{2}\mathcal{L}\{t^2\}$$
$$= \frac{1}{2}\left[\frac{2!}{s^3}\right]$$
$$\mathcal{L}\{a(t)\} = \frac{1}{s^3}$$

7 Sinusoidal function:

$$f(t) = \begin{cases} A \sin\omega t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Sin ωt = ($e^{j\omega t}$ [redacted] $e^{-j\omega t}$) / $2j$

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\mathcal{L}[A\sin\omega t] = \frac{A\omega}{s^2+\omega^2}, \quad \mathcal{L}[A\cos\omega t] = \frac{As}{s^2+\omega^2}$$

complex numbers :-

>> Rectangular form. $\vec{A} = x + jy$

>> exponintial form. $\vec{A} = |\vec{A}| e^{j\theta}$

>> polor form $\vec{A} = |A| \angle\theta$

Euler's law

$\vec{A} = A e^{j\theta} = A\cos\theta + jA\sin\theta$

$x = A\cos\theta \Rightarrow$ $\boxed{\sqrt{x^2+y^2} = A \qquad \tan^{-1}\frac{y}{x} = \theta}$

$y = A\sin\theta$

Properties of Laplace Transforms :-

- $\mathcal{L}\{k f(t)\} = k F(s)$
- $\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
- $\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s) - f(0)$

$\mathcal{L}\left[\frac{d^2f(t)}{dt}\right] = s^2F(s) - sf(0) - f'(0)$

$\mathcal{L}\left[\frac{d^nf(t)}{dt^n}\right] = s^nF(s) - s^{n-1}f(0) - s^{n-2}\frac{df(0)}{dt} - s^{n-3}\frac{d^2f(0)}{dt^2} - \cdots - \frac{d^{n-1}f(0)}{dt^{n-1}}$

$= s^nF(s) - \sum_{k=1}^{n} s^{n-k} f^{(k-1)}(0)$

- $\mathcal{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$

where $f^{(k-1)}(t) = \frac{d^{k-1}}{dt^{k-1}} f(t)$

- $\mathcal{L}[f(t-a)u(t-a)] = e^{-as}F(s)$, $a \geq 0$ [Translation in the time domain]
- $\mathcal{L}[e^{-at}f(t)] = F(s+a)$, $a \geq 0$ [Translation in the freq domain]
- $\mathcal{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$, $a > 0$ [Scale changing]

>> $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = f(\infty)$ [Final value theorem.] ← The theorem is valid only if the poles of F(s), except for a first-order pole at the origin, lie in the left half of the s-plane.

>> $f(0) = \lim_{s\to\infty} sF(s)$. [Initial value theorem] $= \lim_{t\to 0^+} f(t)$

$$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0) \qquad \mathcal{L}[f(t)] = \int_0^{\infty} f(t)\cdot e^{-st}\,dt$$

$$\int_0^{\infty}\left[\frac{d}{dt}f(t)\right]e^{-st}\,dt = sF(s) - f(0)$$

$$\lim_{s\to 0}\int_0^{\infty}\left[\frac{d}{dt}f(t)\right]e^{-st}\,dt = \lim_{s\to 0}\left[sF(s) - f(0)\right]$$

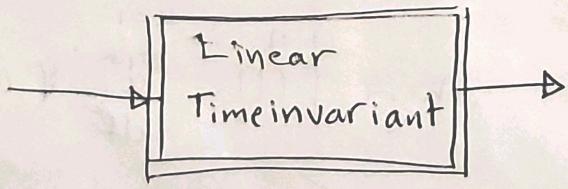
$$= \int_0^{\infty}\left[\frac{d}{dt}f(t)\right]dt = f(t)\Big|_0^{\infty} = f(\infty) - f(0) = \lim_{s\to 0}\left[sF(s) - f(0)\right]$$

$$\Rightarrow \boxed{f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$$ Final value theorem

$$\int_0^{\infty}\left[\frac{d}{dt}f(t)\right]e^{-st}\,dt = sF(s) - f(0)$$

$$\lim_{s\to\infty}\int_0^{\infty}\left[\frac{d}{dt}f(t)\right]e^{-st}\,dt = \lim_{s\to\infty}\left[sF(s) - f(0^+)\right] = 0$$

$$\boxed{f(0^+) = \lim_{s\to\infty} sF(s)}$$



$x_1(t)$ → $y_1(t)$

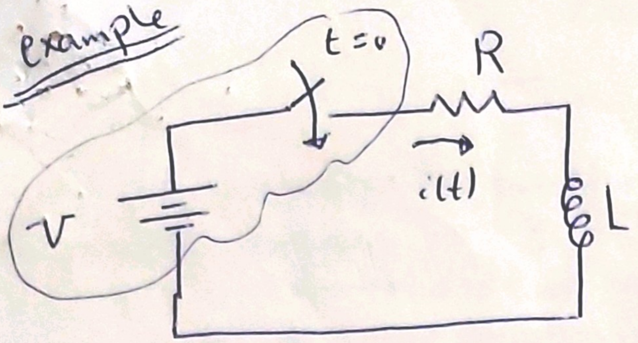
$x_2(t)$ → $y_2(t)$

$x_1(t) + x_2(t)$ ⟶ $y_1(t) + y_2(t)$

$x(t)$ ⟶ $y(t)$

$x(t-t_0)$ ⟶ $y(t-t_0)$

example



$i_L(0) = 0$

constant.

$V u(t) = R\, i(t) + L \frac{di(t)}{dt}$

$V \cdot \frac{1}{s} = R\, I(s) + L\left(s I(s) - i_L(0^-)\right)$

$\frac{V}{s} = (R + Ls)\, I(s)$

$I(s) = \frac{V}{s} \cdot \frac{1}{R+Ls}$

$I(s) = \frac{V/L}{s\left(s+\frac{R}{L}\right)} = \frac{K_1}{s} + \frac{K_2}{s+\frac{R}{L}}$

$K_1\left(s+\frac{R}{L}\right) + K_2(s) = \frac{V}{L}$

(*) $s = 0 \Rightarrow K_1\left(\frac{R}{L}\right) = \frac{V}{L}$

$\boxed{K_1 = \frac{V}{R}}$

(*) $s = -\frac{R}{L}$

$K_2\left(-\frac{R}{L}\right) = \frac{V}{L}$

$\boxed{K_2 = -\frac{V}{R}}$

$i(t) = \frac{V}{R} u(t) - \frac{V}{R} e^{-\frac{R}{L}t} u(t)$

$i(t) = \frac{V}{R}\left[1 - e^{-R/L\, t}\right] u(t)$

KCL

$\Sigma\, i(t) = 0$

$\Sigma\, I(s) = 0$

KVL

$\Sigma\, v(t) = 0$

$\Sigma\, V(s) = 0$

Chapter 13

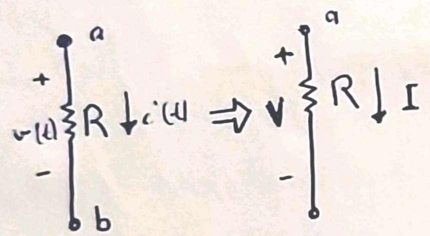
» The Laplace transform has two characteristics that make it an attractive tool in circuit analysis.

- First, it transforms a set of linear constant-coefficient differential equations into a set of linear polynomial equations, which are easier to manipulate.
- Second, it automatically introduces into the polynomial equations the initial values of the current and voltage variables. Thus, initial conditions are an inherent part of the transform process. (This contrasts with the classical approach to the solution of differential equations, in which initial conditions are considered when the unknown coefficients are evaluated.)

$s=\sigma+j\omega$

Circuit Elements in the s Domain :-

⊛ Resistor in the s Domain



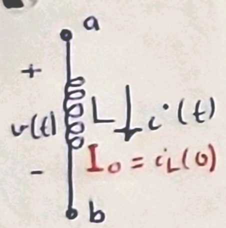
$v = Ri$

$V = RI$ (R → constant)

$V(s) = R\cdot I(s)$

$$\boxed{\frac{V(s)}{I(s)} = R}$$

⊛ Inductor in the s Domain

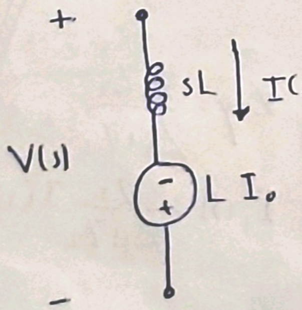


$v = L\dfrac{di}{dt}$

$V(s) = L[sI(s) - i_L(0)]$

$V(s) = sL\,I(s) - L I_0 \Longrightarrow Z_L(s) = \dfrac{V(s)}{I(s)}\Big|_{i_L(0)=0}$

$$\boxed{Z_L(s) = sL}$$

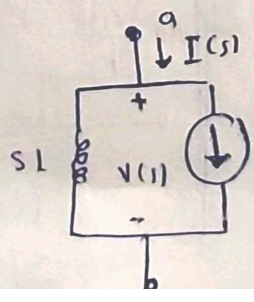


note that :-

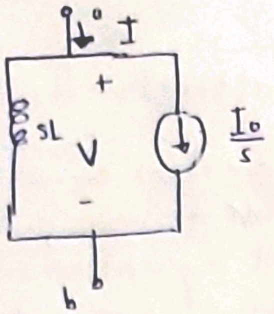
* $Z(s) \equiv \text{Impedance} = \dfrac{V(s)}{I(s)}$
* $Y(s) = \text{admittance} = \dfrac{I(s)}{V(s)} = \dfrac{1}{Z(s)}$

From Source Transformation →

From Current Inductor Law →



$\dfrac{I_0 L}{sL} = \dfrac{I_0}{s}$



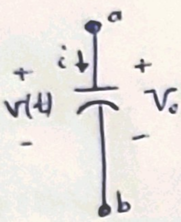
$$i_L(t) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} v(t)\,dt$$

$$I(s) = \frac{I_0}{s} + \frac{1}{L}\left[\frac{V(s)}{s}\right]$$

$$= \frac{I_0}{s} + \frac{V(s)}{sL}$$

$$\boxed{I(s) = I_1(s) + I_2(s)}$$

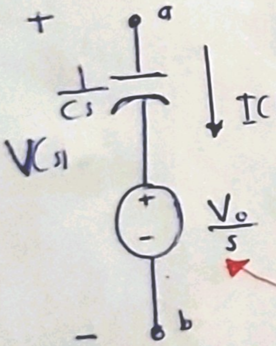
⊛ Capacitor in the s Domain



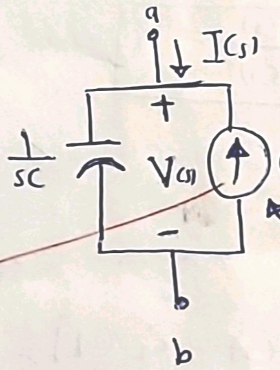
$$v(t) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i(t)\,dt$$

$$V(s) = \frac{V_0}{s} + \frac{1}{C}\,\frac{I(s)}{s}$$

$$\boxed{Z_c(s) = \left.\frac{V(s)}{I(s)}\right|_{V_0=0} = \frac{1}{Cs}}$$



$\Longrightarrow$



$$C V_0 = \frac{V_0}{s} \cdot \frac{1}{\cancel{sC}} * \frac{sC}{1}$$

$$= V_0 C \quad \text{ok}$$

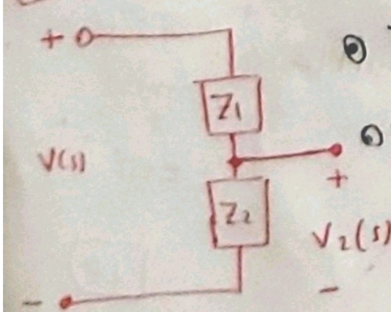
or

$$i(t) = C\,\frac{dv(t)}{dt}$$

$$I(s) = C\left[sV(s) - V_0\right]$$

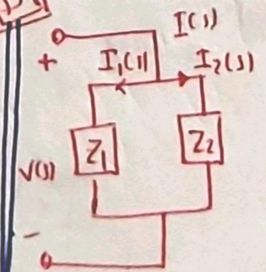
$$\boxed{I(s) = Cs\,V(s) - CV_0}$$

[A]



- $Z_{eq} = Z_1 + Z_2$
- $V_2(s) = \dfrac{Z_2}{Z_1+Z_2}\,V(s)$

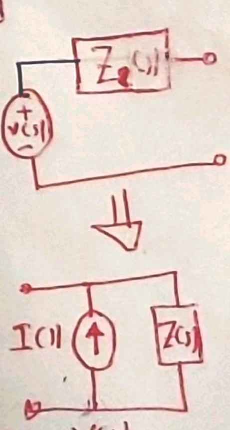
[B]



$$Z_{eq} = \frac{Z_1 Z_2}{Z_1+Z_2}$$

$$I_1(s) = \frac{Z_2}{Z_1+Z_2}\,I(s)$$

[C]



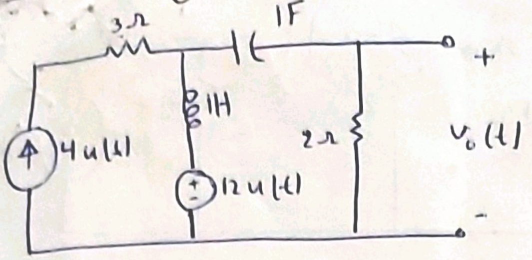
$$I(s) = \frac{V(s)}{Z(s)}$$

[D]

$$\sum I(s) = 0$$

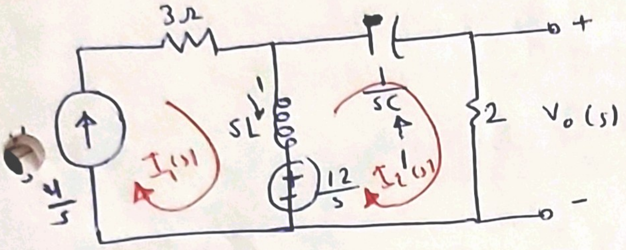
$$\sum V(s) = 0$$

mesh analysis :- Mesh Analysis :-



Find $v_o(t)$:-

① Transformed circuit to s-domain :-



mesh ①

(اجباری عوض)

$I_1(s) = \frac{4}{s}$ Con strain eq ——— ①

mesh ②

$\left(s + \frac{1}{s} + 2\right) I_2(s) - s\, I_1(s) = \frac{12}{s}$ ——— ②

$V_o(s) = 2 \boxed{I_2(s)}$

we need to find $I_2(s)$

from eq ②

$\left(s + \frac{1}{s} + 2\right) I_2(s) - s\left(\frac{4}{s}\right) = \frac{12}{s}$ ← I_1

$$\boxed{I_2(s) = \frac{4(3+s)}{(s+1)^2}}$$

but

using partial fraction

$$V_o(s) = \frac{8(3+s)}{(s+1)^2} = \frac{k_1}{s+1} + \frac{k_2}{(s+1)^2}$$

$k_1(s+1) + k_2 = 8(3+s)$

$\boxed{s=-1} \Rightarrow k_2 = 8(3-1) = 16$

$\boxed{k_2 = 16}$

$$= \frac{8}{s+1} + \frac{16}{(s+1)^2}$$

$\boxed{\text{diff}}$

$\boxed{k_1 = 8}$

or

$k_1(s+1) + 16 = 8(3+s)$

$\boxed{s=0}$

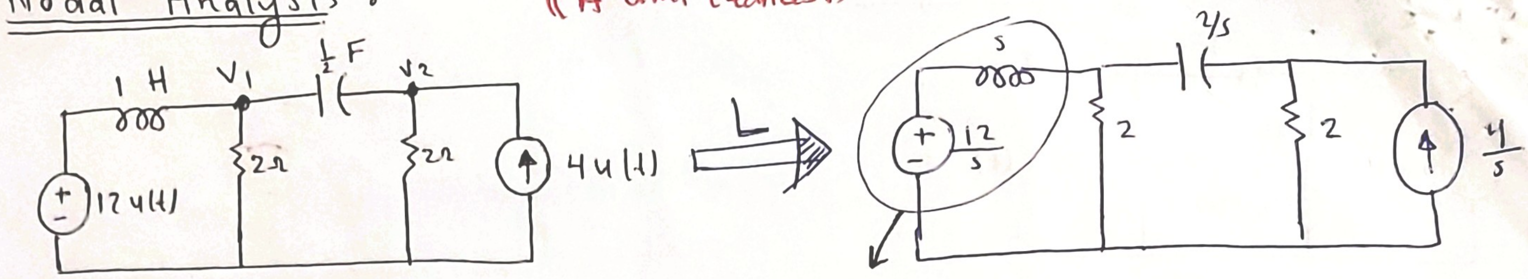
$k_1 + 16 = 24$

$\boxed{k_1 = 8}$

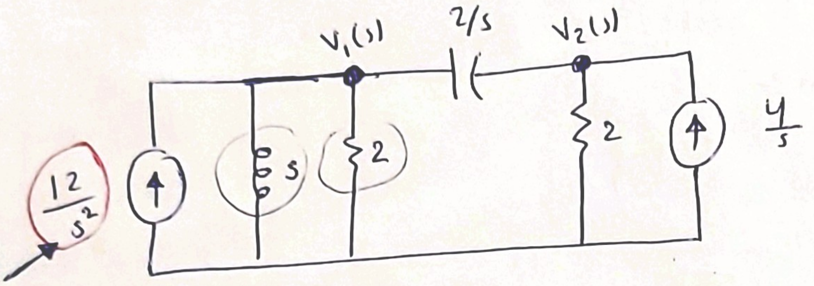
$$\boxed{t^n e^{at} \Rightarrow \frac{n!}{(s-a)^{n+1}}} = \left(8e^{-t} + 16te^{-t}\right)u(t)$$

Nodal Analysis :-

((Admittances))



source transformation.



at node ①

admittance

$$\left(\frac{1}{s} + \frac{1}{2} + \frac{s}{2}\right) V_1(s) - \frac{s}{2} V_2(s) = \frac{12}{s^2} \quad \text{——— ①}$$

at node ②

$$\left(\frac{1}{2} + \frac{s}{2}\right) V_2(s) - \frac{s}{2} V_1(s) = \frac{4}{s}$$

$V = RI$

$I = \frac{1}{R} V$

$\boxed{I = Y\, V(s)}$ ok

partial fraction

$$V_1(s) = \frac{4(s^2 + 3s + 3)}{s(s^2 + \frac{3}{2}s + 1)} = \frac{A}{s} + \frac{k + k^*}{[\;]\;(\;)}$$

$$\mathcal{L}^{-1}\left[\frac{k}{s+a-j\beta} + \frac{k^*}{s+a+j\beta}\right]$$

$$= 2|k|\, e^{-at} \cos(\beta t + \theta)$$

$k = |k| e^{j\theta} = |k| \angle\theta^\circ$

$k^* = |k| e^{-j\theta} = |k| \angle -\theta$

partial fraction coefficient.

$$\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$$

$$V_2(s) = \frac{4(s^2 + 3s + 2)}{s(s^2 + \frac{3}{2}s + 1)}$$

$$v_1(t) = \left(12 + 8e^{-3/4\, t} \cos\left[\left(\frac{\sqrt{7}}{4}\right)t + 180^\circ\right]\right) u(t)$$

$\boxed{\text{at } t = \infty} \Rightarrow (v_1(t) = 12)$

using **Cramer's** method to solve for V_1 and V_2

Review :-

$$\left(\frac{1}{s} + \frac{1}{2} + \frac{s}{2}\right) v_1(s) - \frac{s}{2} V_2(s) = \frac{12}{s^2}$$

$$-\frac{s}{2} v_1(s) + \left(\frac{1}{2} + \frac{s}{2}\right) V_2(s) = \frac{4}{s}$$

$$\Delta = \begin{vmatrix} \frac{2+s+s^2}{2s} & -\frac{s}{2} \\ -\frac{s}{2} & \frac{(1+s)}{2} \end{vmatrix}$$

$\boxed{V_1 = \frac{N_1}{\Delta},\; V_2 = \frac{N_2}{\Delta}}$

$$N_1 = \begin{vmatrix} \frac{12}{s^2} & -\frac{s}{2} \\ 4/s & \frac{1+s}{2} \end{vmatrix}, \quad N_2 = \begin{vmatrix} \frac{2+s+s^2}{2s} & \frac{12}{s} \\ -s/2 & 4/s \end{vmatrix}$$

$$V_1(s) = \frac{4(s^2+3s+3)}{s(s^2+\frac{3}{2}s+1)} = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + \dots + b_1 s + b_0}.$$

* if $m > n$ proper rational function.
* if $m \leq n$ improper rational function.

$$\frac{N(s)}{D(s)} = \frac{K(s+z_1)(s+z_2)\dots(s+z_n)}{(s+p_1)(s+p_2)\dots(s+p_m)}.$$

- Distinct Real Roots of D(s).
- Distinct Complex Roots of D(s).
- Repeated Real Roots of D(s).
- Repeated Complex Roots of D(s).

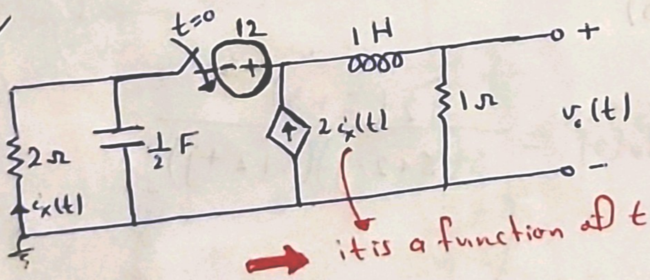
notes:-
② in time
* Translation ① in frequency ① $e^{-at} f(t) \longrightarrow F(s+a)$, ② Time $f(t-a)u(t-a) \rightarrow e^{-as}F(s)$
* exponintial form $\vec{A} = |\vec{A}| e^{j\theta}$, $|A| = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\frac{y}{x}$, $\vec{A} = x + jy$
* $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$
* $\frac{du(t)}{dt} = \delta(t)$, $\delta(t) \overset{\mathcal{L}}{\Longrightarrow} 1$

$\frac{d\delta(t)}{dt} \overset{\mathcal{L}}{\longrightarrow} s$

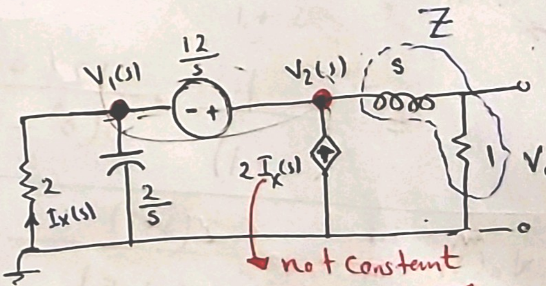
$\frac{d^2\delta(t)}{dt^2} \overset{\mathcal{L}}{\longrightarrow} s^2$

$\delta(t-t_0) \overset{\mathcal{L}}{\longrightarrow} e^{-t_0 s} \cdot \boxed{F(s)}$ (1) $= e^{-t_0 s}$ $\Longrightarrow$ Go to page 17

example
super node



it is a function of t



not constant

$i_x(t) \longrightarrow$	$I_x(s)$
$f(t) \longrightarrow$	$F(s)$

$V_2(s) - V_1(s) = \frac{12}{s}$ (constrain eq)

Y admitt ʊ
$(\frac{s}{2} + \frac{1}{2})V_1(s) + \frac{1}{s+1}V_2(s) = 2I_x(s)$ (super-node equation)

but;
$\boxed{I_x(s) = -\frac{V_1(s)}{2}}$ and $\boxed{V_o(s) = \frac{1}{s+1}V_2(s)}$ ⊙ using voltage divider

$$\left(\frac{s}{2}+\frac{1}{2}\right)V_1(s)+\frac{1}{s+1}V_2(s)=2I_x(s)$$

$$V_1=V_2(s)-\frac{12}{s}\quad,\quad I_x=-\frac{V_1(s)}{2}$$

$$\left(\frac{s}{2}+\frac{1}{2}\right)V_1(s)+\frac{1}{s+1}V_2(s)=-2\left(\frac{V_1(s)}{-2}\right)$$

$$\frac{V_2(s)}{s+1}=\left[V_1(s)-\left(\frac{s}{2}+\frac{1}{2}\right)V_1(s)\right]$$

$$\frac{V_2(s)}{s+1}=-V_1(s)\left[\frac{1}{2}+\frac{s+1}{2}\right]=-\left(\frac{s+3}{2}\right)\left(V_2-\frac{12}{s}\right)$$

$$\frac{V_2(s)}{s+1}+\left(\frac{s+3}{2}\right)(V_2)=+\left(\frac{s+3}{2}\right)\left(\frac{12}{s}\right)$$

$$V_2(s)\left[\frac{2}{2(s+1)}+\frac{(s+1)(s+3)}{(s+1)(2)}\right]=\left(\frac{s+3}{2}\right)\left(\frac{12}{s}\right)\left(\frac{2(s+1)}{(s+1)(s+3)+2}\right)$$

$$s^2+4s+3+2$$

$$V_2(s)=\frac{12(s+3)(s+1)}{s(s^2+4s+5)}$$

$$\therefore\ V_o(s)=\frac{1}{s+1}\cdot V_2(s)$$

$$=\frac{1}{(s+1)}\cdot\frac{12(s+3)(s+1)}{s(s^2+4s+5)}$$

$$V_o(s)=\frac{12(s+3)}{s(s^2+4s+5)}$$

$$\frac{-4\pm\sqrt{16-4(1)(5)}}{2(1)}$$

$$s_{1,2}=-2\pm j$$

$$s_{1,2}=\frac{-4\pm j2}{2}=-2\pm j$$

$$V_o(s)=\frac{12(s+3)}{s(s+2-j)(s+2+j)}$$

$$V_o(s)=\frac{K_1}{s}+\frac{K_2}{s+2-j}+\frac{K_2^*}{s+2+j}$$

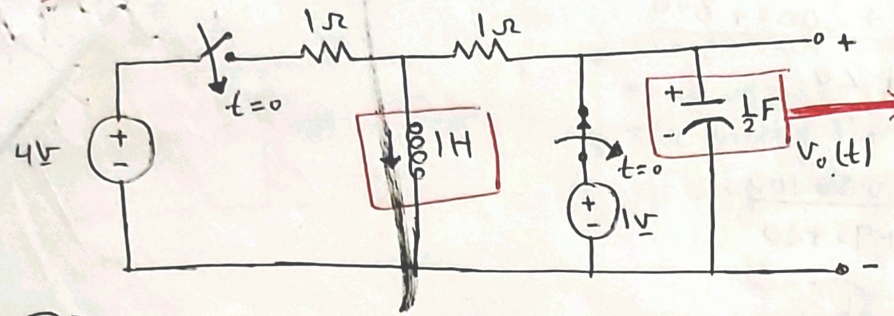
$$K_2=3.79\angle 161.57$$

α ↑2, B ↑1

$$2|k|e^{-\alpha t}\cos(Bt+\theta)$$

$$V_o(t)=\left[7.2+7.58e^{-2t}\cos(t+161.57)\right]u(t)\ \ V$$

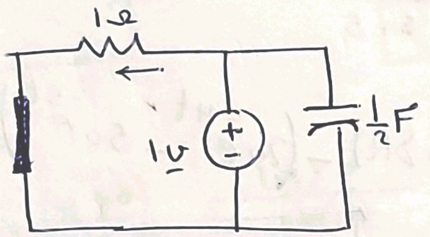
example elements with Initial Conditions:-



there is two models For each one
→ C
→ L
when there is initial condition.

?! Find $v_o(t)$ for $t>0$:-

for $t<0$



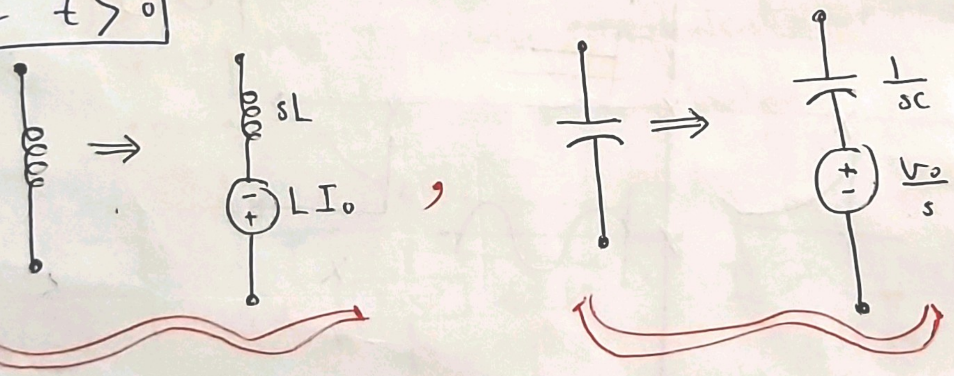
$i_L(0^-) = 1A$

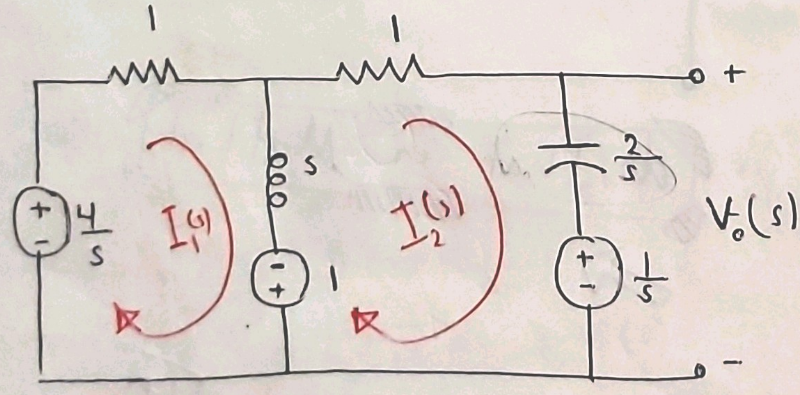
$v_c(0^-) = 1V$

$v = L\frac{di}{dt}$

$v = 0 \Rightarrow$ short ckT

for $t>0$





$\Rightarrow$ Find $I_2(s)$ $\Rightarrow$ $V_o(s) = \frac{2}{s} I_2(s) + \frac{1}{s}$

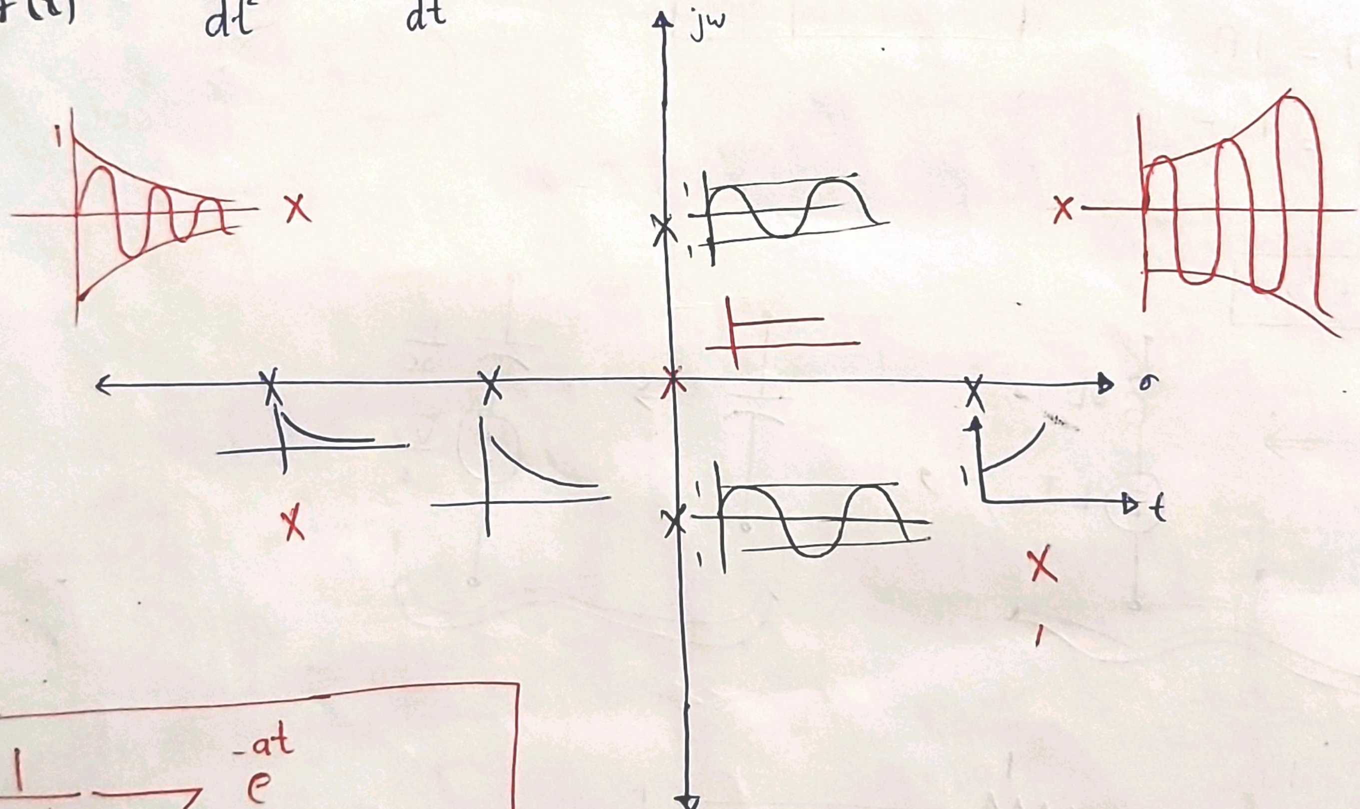
example

$$F(s) = \frac{s^4 + 13s^3 + 66s^2 + 200s + 300}{s^2 + 9s + 20}$$

$$= \underbrace{s^2 + 4s + 10}_{\text{الناتج}} + \underbrace{\frac{30s + 100}{s^2 + 9s + 20}}_{\text{الباقي}}$$

$$= s^2 + 4s + 10 - \frac{20}{s+4} + \frac{50}{s+5}$$

$$f(t) = \frac{d^2\delta(t)}{dt^2} + 4\frac{d\delta(t)}{dt} + 10\delta(t) - \left(20e^{-4t} - 50e^{-5t}\right)u(t)$$



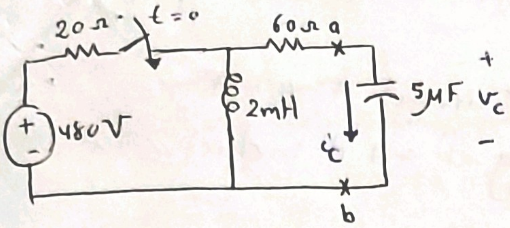
$$\frac{1}{s+a} \longrightarrow e^{-at}$$

$$\frac{w}{s^2+w^2} \longrightarrow \sin wt$$

$$\frac{w}{(s+a)^2+w^2} \longrightarrow e^{-at}\sin wt$$

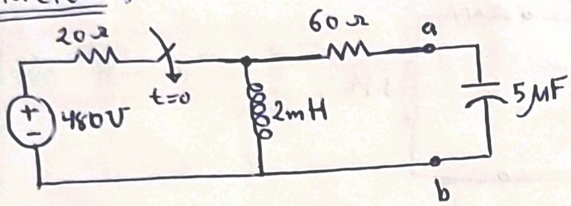
do that in MatLab

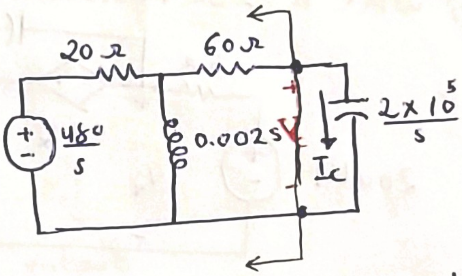
The Use of Thevenin's Equivalent :-



- The problem is to find the capacitor current that results from closing the switch.
- The energy stored in the circuit prior to closing is zero.

Solution:





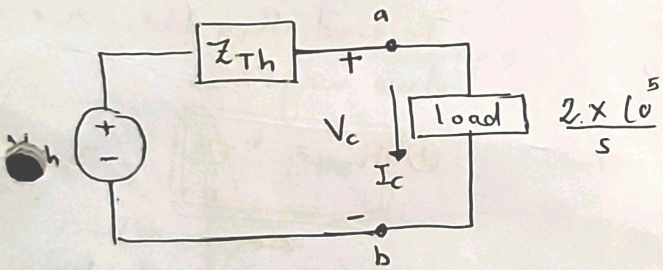
V_{TH} (load open ckT)
Z_{TH} (open ckT + kill sources)

Voltage divider

$$V_{Th} = \frac{0.002s}{20+0.002s}\cdot\left(\frac{480}{s}\right) = \frac{480}{s+10^4}$$

$$Z_{Th} = 60 + (0.002s \parallel 20) = 60 + \frac{0.002s(20)}{20+0.002s} = \frac{80(s+7500)}{s+10^4}$$

⟹ Using the Thevenin equivalent, we reduce the circuit shown above to the one shown below:-



$\frac{2\times10^5}{s}$

$$I_c = \frac{V_{TH}}{Z_{TH}+Z_{load}} = \frac{6s}{(s+5000)^2}$$

$$I_c = \frac{-30,000}{(s+5000)^2} + \frac{6}{s+5000}$$

$$\mathcal{L}^{-1}[I_c] = i_c = (-30,000\,t e^{-5000t} + 6e^{-5000t})\,u(t)\ \text{A}.$$

Two ways!

V_c ?!
1. find $V_c \to v_c$
2. $v_c = \frac{1}{C}\int_{0^-}^{t} i_c\,dt + v_c(0^-)$

$$V_c = \frac{1}{sC} I_c = \frac{2\times10^5}{s}\cdot\frac{6s}{(s+5000)^2} = \frac{12\times10^5}{(s+5000)^2}$$ direct.

$$v_c = 12\times10^5\, t e^{-5000t}\, u(t).$$

or

$$v_c = 2\times10^5 \int_{0^-}^{t} \underbrace{(6-30,000t)e^{-5000t}}_{i_c(t)}\,dt$$

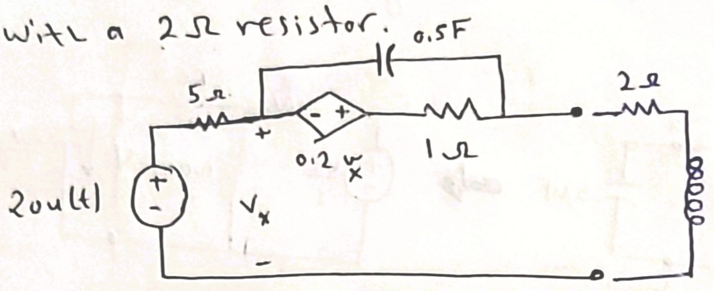
$V = ZI$

example

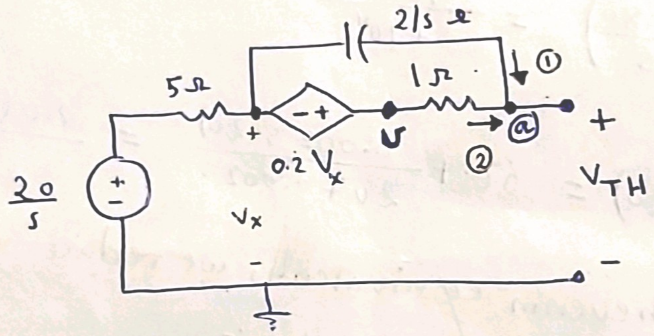
The initial charge on the capacitor in the circuit shown is zero.

(a) Find the s-domain Thevenin eq. CKT. with respect to terminals a and b.

(b) Find the s-domain expression for the current that the circuit delivers to a load consisting of a 1 H inductor in series with a 2Ω resistor.



$V = ZI$



$v = 0.2V_x + V_x = 1.2V_x$

$= 1.2\left(\frac{20}{s}\right)$

With no load across terminals a-b, $V_x = \frac{20}{s}$

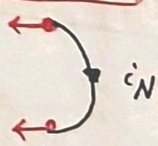
KCL @ node ⓐ

$$\underbrace{\left(\frac{20}{s} - V_{TH}\right)\frac{s}{2}}_{①} + \underbrace{\left(1.2\left(\frac{20}{s}\right) - V_{TH}\right)1}_{②} = 0$$

$$V_{TH} = \frac{20(s+2.4)}{s(s+2)}$$

To find R_{TH} there are two ways:-

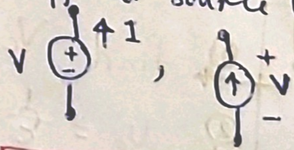
① $R_{TH} = \frac{V_{TH}}{I_N}$



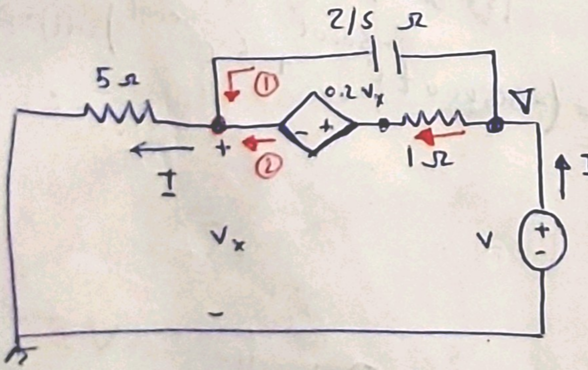
without killing the sources.

② ⓐ indep sources = 0

ⓑ applied source (ـتـ)



$\frac{V}{I} = R_{TH}$

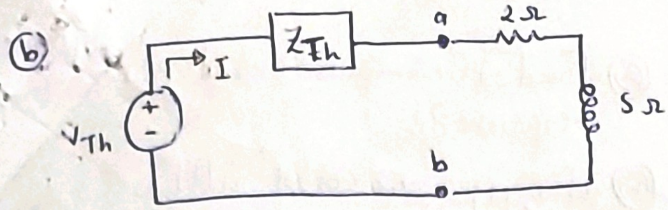


$V_x = 5I$ and $Z_{TH} = \frac{V}{I}$

$$I = \overset{①}{(V - V_x)\cdot\frac{s}{2}} + (V - 1.2V_x)(1)$$

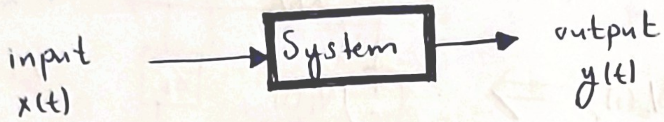
$$I = (V - 5I)\cdot\frac{s}{2} + V - 6I \Rightarrow Z_{th} = \frac{5(s+2.8)}{s+2}$$

(b)



$$I = \frac{V_{TH}}{Z_{Th}+2+s} = \frac{20(s+2.4)}{s(s+3)(s+6)}.$$

The Transfer Function :



- The transfer function is defined as the s-domain ratio of the Laplace transform of the output (response) to the Laplace transform of the input (source).
In computing the transfer function, we restrict our attention to circuits where all initial conditions are zero.
If a circuit has multiple independent sources, we can find the transfer function for each source and use superposition to find the response to all sources.

$$H(s) = \left.\frac{Y(s)}{X(s)}\right|_{\text{initial condits} = 0}, \quad Y(s) = H(s)\cdot X(s)$$

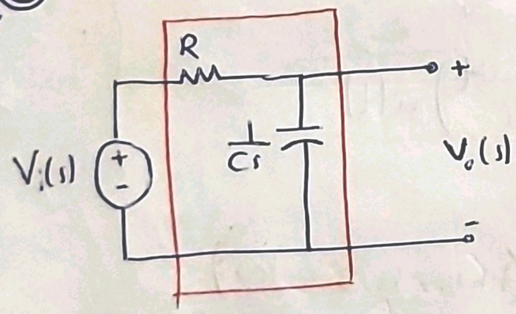
Y(s) → output. → response.

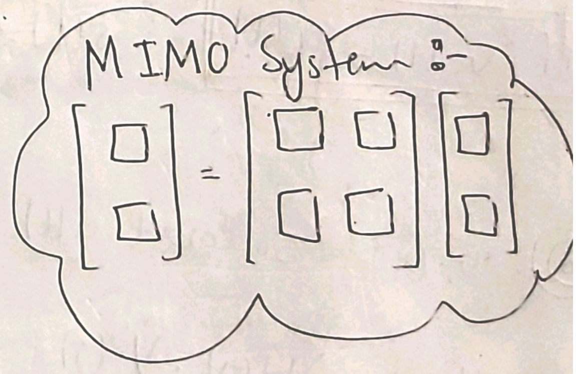
① if $X(s) = \delta(t)$ → impulse resp
② if $X(s) = u(t)$ → step resp
③ if $X(s) = r(t)$ → ramp resp.

① $\mathcal{L}^{-1}[H(s)] = h(t) \equiv$ impulse resp.

② $\mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = s(t) \equiv$ unit step resp.

Example ①





$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{1/Cs}{1/Cs + R}\cdot V_i(s)$$

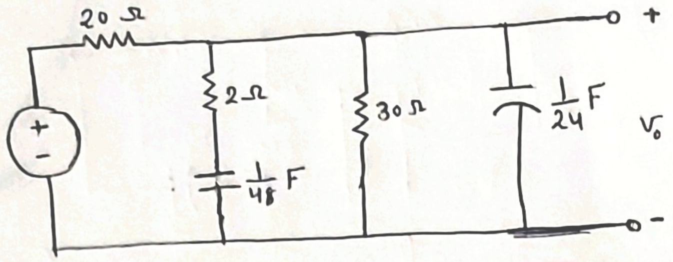
$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{s+1}}$$ ← (order of the system)

the same of output since $V_i(s) = 1$

$$V_o(s) = \left(\frac{1}{s+1}\right)(1) = H(s)$$

$$v_o(t) = h(t) = \mathcal{L}^{-1}[H(s)] = \text{Impulse response} = e^{-t}u(t)$$

example ③ go to example ② →

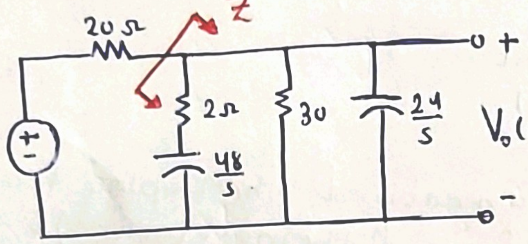


ⓐ Find the impulse and unit-step response ?!

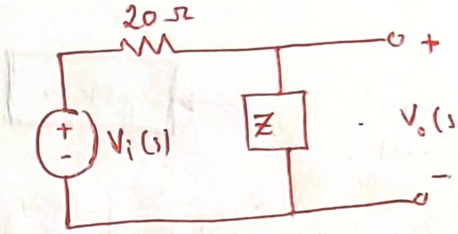
ⓑ If $v_i(t) = 50\cos 2t \; u(t)$
Find $v_o(t)$?!

$$H(s) = \frac{V_o(s)}{V_i(s)}\Bigg|_{i.c=0}$$

example ② in the next page →



⟹



$$Z = \frac{24}{s} \,\|\, 30 \,\|\, \left(2 + \frac{48}{s}\right)$$

$$V_o(s) = \frac{Z}{Z+20} V_i(s) \;\Rightarrow\; H(s) = \frac{Z}{Z+20} = \frac{V_o(s)}{V_i(s)}$$

$$H(s) = \frac{1.2(s+2)}{(s+1)(s+4)}$$

← zeros. (numerator)
← poles. (denominator)

if $v_i(t) = \delta(t)$

$$v_o(t) = h(t) = \mathcal{L}^{-1}[H(s)]$$

$$h(t) = \left(0.4\, e^{-t} + 0.8\, e^{-4t}\right) u(t)$$

if $v_i(t) = u(t)$

$$s(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right]$$ ← (there is 3 poles.)

ⓑ $v_i(t) = 50\cos 2t \; u(t)$

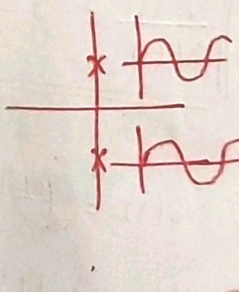
$$V_o(s) = H(s) \cdot V_i(s)$$

$$= \frac{1.2(s+2)}{(s+1)(s+4)} \cdot 50 \frac{s}{s^2+4}$$

$\Rightarrow$ complex poles (sin, cosine)

T.F من طبيعة الدائرة

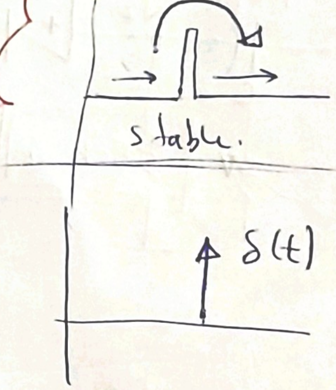
natural transient resp. | steady-state response. → من المصادر sources



$\therefore v_o(t) = \left[+4e^{-t} - 8e^{-4t} + (12\cos 2t + 12\sin 2t) \right] u(t)$

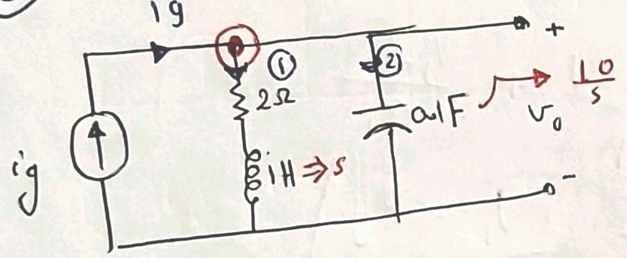
- $+4e^{-t} - 8e^{-4t}$: come frome poles $(s+1)(s+4)$ goes to zero aftertime
- $(12\cos 2t + 12\sin 2t)$: Since there is only imagin poles — forced (steady-state) response. $\Rightarrow$ poles of input signal only.

The same state.



transien respons = complet response.

Example ②



(a) Derive the numerical expression for the transfer function V_o/I_g for the circuit shown.

(b) Give the numerical value of each pole and zero of $H(s)$.

[a] $I_g = \dfrac{V_o}{s+2} + \dfrac{V_o s}{10} \Rightarrow I_g = V_o\left(\dfrac{1}{s+2} + \dfrac{s}{10}\right)$

$= V_o\left(\dfrac{10 + s(s+2)}{(s+2)10}\right)$ ← ok

$$\frac{V_o}{I_g} = H(s) = \frac{10(s+2)}{s^2+2s+10}$$

Zeros $s = -2$

poles $s^2 + 2s + 10 = 0$ ← Characteristic equation

$(s+1+3j)(s+1-3j) = 0$

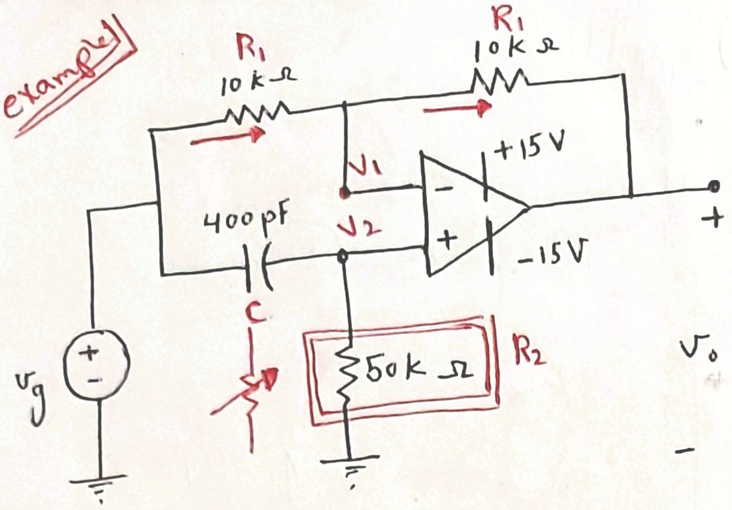
$$s_{1,2} = -1 \pm 3j$$

** stable

BIBO

$\dfrac{-2 \pm \sqrt{4 - 4(1)(10)}}{2(1)} = \dfrac{-2 \pm \sqrt{-36}}{2(1)} = \dfrac{-2 \pm j6}{2} = -1 \pm 3j$

example



(a) For the CKT shown, find the steady-state expression for v_o when $v_g = 10\cos 50{,}000t$ V

(b) Replace the 50kΩ resistor with a variable resistor and compute the value of resistance necessary to cause v_o to lead v_g by 120.

① $V_1 = V_2 = \frac{R_2}{R_2 + \frac{1}{sC}} \cdot V_g$

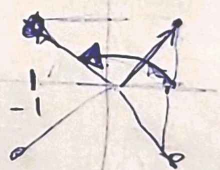
② $\frac{V_g - V_1}{R_1} = \frac{V_1 - V_o}{R_1} \Rightarrow \frac{V_1 - V_g}{R_1} + \frac{V_1 - V_o}{R_1} = 0$

$2V_1 - V_g - V_o = 0 \Rightarrow$ $\boxed{V_o = 2V_1 - V_g}$

$V_o = 2(V_1) - V_g$

$\Downarrow$? $V_g \Longrightarrow \frac{V_o}{V_g} = H(s)$

$\frac{A e^{-j135}}{A}$



$H(s) = \frac{R_2 C s - 1}{R_2 C s + 1} = \frac{2*10^{-5} s - 1}{2*10^{-5} s + 1}$

$H(j\omega) = H(j50{,}000) = \frac{j-1}{j+1} = \frac{A e^{j(135)}}{A e^{45}}$

$\boxed{\frac{j-1}{j+1} * \frac{1-j}{1-j}}$

$= e^{j90} = 1\angle 90°$

or $= j = 1\angle 90$

$y_{ss}(t) = A|H(j\omega)|\cos[\omega t + \phi + \phi(\omega)]$

$v_o = 10(1)\cos(50{,}000t + 0 + 90)$

$\boxed{v_o = 10\cos(50{,}000t + 90)}$ V